

Robust Optimization of the Shape of Permanent Magnets in a Synchronous Machine Considering Stochastic Quantities

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A permanent magnet (PM) synchronous machine is robustly optimized according to the parameters defining the size and the position of the PMs. In a deterministic setting the worst case deviation on these parameters is considered. This approach needs first and second order derivatives. The second setting makes use of uncertainty quantification accounting for the standard deviation. In this case only first order derivatives are needed. The PM volume is successfully reduced and it is shown that both settings are equivalent when applying linearization.

Index Terms—Finite element analysis, gradient methods, monte carlo methods, permanent magnet machines.

I. INTRODUCTION

THE PERMANENT MAGNETS (PMs) of a 3-phase 6-pole PM synchronous machine (PMSM) are subjected to optimization. The goal is to reduce the size of the PM material while accounting for the deviations that might occur on the parameters describing the size and the position of the PMs. In this paper, a deterministic optimization method using gradients is applied in order to guarantee a small number of iteration steps. The deterministic method belongs to the class of direct optimization methods [1].

II. MODEL

A full description of the machine can be found in [2]. The width and height of the PM is depicted by p_1 and p_2 respectively. The PMs are buried in the rotor at a depth p_3 . These quantities should be contained in the admissible set $\mathcal{P}_{\text{ad}} = \{\mathbf{p} \in \mathbb{R}^3 | G(\mathbf{p}) \leq 0\}$, where $G(\mathbf{p})$ represents the constraints on $\mathbf{p} = (p_1, p_2, p_3)$. The machine is subjected to an optimization in which the amount of PM material is minimized while maintaining a prescribed EMF E_d and accounting for deviations on \mathbf{p} .

The PMSM is described by solving the magnetostatic approximation of the Maxwell equations. Using the magnetic vector potential $\vec{A}(x, y, z, \mathbf{p})$, one has to solve the partial differential equation (PDE)

$$\vec{\nabla} \times \left(\nu(\mathbf{p}) \vec{\nabla} \times \vec{A}(\mathbf{p}) \right) = \vec{J}_{\text{src}} - \vec{\nabla} \times \vec{H}_{\text{pm}}(\mathbf{p}), \quad (1)$$

with adequate boundary conditions. The reluctivity is denoted by $\nu(\mathbf{p}) = \nu(x, y, z, \mathbf{p})$, the source current density by $\vec{J}_{\text{src}}(x, y, z)$ and the PM's source magnetic field strength by $\vec{H}_{\text{pm}}(x, y, z, \mathbf{p})$. The ansatz $\vec{A}(\mathbf{p}) \approx \sum_{j=1}^{N_D} a_j(\mathbf{p}) \vec{w}_j(x, y) = \sum_{j=1}^{N_D} a_j(\mathbf{p}) N_j(x, y) / \ell_z \vec{e}_z$, with N_D the total number of degrees of freedom, $\vec{w}_j(x, y)$ the edge shape functions related to the nodal finite elements $N_j(x, y)$ and ℓ_z the length of the machine, is used. Using the Galerkin approach in the 2D planar case leads to the system of equations $\mathbf{K}_\nu(\mathbf{p}) \mathbf{a}(\mathbf{p}) = \mathbf{j}_{\text{src}} + \mathbf{j}_{\text{pm}}(\mathbf{p})$ [3]. $\mathbf{K}_\nu(\mathbf{p})$ is the finite element system matrix, $\mathbf{a}(\mathbf{p})$

are the unknowns. The right hand side depicts the discretized right hand side of (1). Solving this system and applying the loading method [4] enables to calculate of the EMF.

The parameters \mathbf{p} are uncertain due to the production process. It is assumed that $\mathbf{p}(\omega) = \bar{\mathbf{p}} + \delta'(\omega)$. The stochastic nature of a quantity is depicted by ω and $\bar{\mathbf{p}} = \mathbb{E}[\mathbf{p}(\omega)]$ is the expectation value. The $\delta'(\omega)$ are independently and uniformly distributed random variables:

$$\delta'(\omega) \sim \mathcal{U}(\delta^l, \delta^u). \quad (2)$$

In our numerical experiments $-\delta^l = \delta^u = \Delta$. The value of Δ is increased from 0 to 0.2 mm.

III. ROBUST OPTIMIZATION PROCEDURE

First the deterministic setting without deviations (D Opt) is considered. Let J_1 depict the cost function so that

$$\min_{\bar{\mathbf{p}} \in \mathbb{R}^3} J_1(\bar{\mathbf{p}}) := \bar{p}_1 \bar{p}_2 \quad (3)$$

subject to the constraints $G_1(\bar{\mathbf{p}}, \mathbf{a}(\bar{\mathbf{p}})) \leq 0$. This optimization problem can be solved by a standard method. In this work Sequential Quadratic Programming (SQP) with damped Broyden–Fletcher–Goldfarb–Shanno (BFGS) updates for the Hessian approximation [6] is used.

Due to the uncertainty a worst-case robust counterpart (D Rob 1) for (3) is introduced by considering

$$\min_{\bar{\mathbf{p}} \in \mathbb{R}^3} \max_{\delta \in U} J_1(\bar{\mathbf{p}} + \delta), \quad (4a)$$

subject to

$$\max_{\delta \in U} G_1(\bar{\mathbf{p}} + \delta, \mathbf{a}(\bar{\mathbf{p}})) \leq 0, \quad (4b)$$

with the uncertainty set $U := \{\delta \in \mathbb{R}^3 | \|D^{-1} \delta\|_\infty \leq 1\}$, and $D = \text{diag}(\Delta, \Delta, \Delta)$. This nested optimization problem is hard to solve. Hence, an approximation of the max problem is utilized. Since the deviations are small a local linearization can be applied, see e.g. [7], so that a numerically feasible optimization problem is obtained. For this purpose, the first

order Taylor approximations of the cost function and the constraint are considered according to $\bar{\mathbf{p}}$:

$$J_1(\bar{\mathbf{p}} + \delta) \approx J_1(\bar{\mathbf{p}}) + \nabla_{\bar{\mathbf{p}}} J_1(\bar{\mathbf{p}}) \delta \quad (5)$$

$$G_1^{(i)}(\bar{\mathbf{p}} + \delta, \mathbf{a}(\bar{\mathbf{p}})) \approx G_1^{(i)}(\bar{\mathbf{p}}, \mathbf{a}(\bar{\mathbf{p}})) + \nabla_{\bar{\mathbf{p}}} G_1^{(i)}(\bar{\mathbf{p}}, \mathbf{a}(\bar{\mathbf{p}})) \delta, \quad (6)$$

for $i = 1, \dots, N_G$, where N_G is the number of constraints. Inserting this approximation in (4a), one obtains the linear approximation of the robust counterpart:

$$\min_{\bar{\mathbf{p}} \in \mathbb{R}^3} J_2 := J_1(\bar{\mathbf{p}}) + \|D \nabla_{\bar{\mathbf{p}}} J_1(\bar{\mathbf{p}})\|_1, \quad (7a)$$

subject to

$$G_2 := G_1^{(i)}(\bar{\mathbf{p}}, \mathbf{a}(\bar{\mathbf{p}})) + \|D \nabla_{\bar{\mathbf{p}}} G_1^{(i)}(\bar{\mathbf{p}}, \mathbf{a}(\bar{\mathbf{p}}))\|_1 \leq 0, \quad (7b)$$

for $i = 1, \dots, N_G$. The dual norm $\|\cdot\|_*$ is defined as

$$\|\cdot\|_* : \mathbb{R}^n \rightarrow \mathbb{R} \\ \mathbf{g} \mapsto \|\mathbf{g}\|_* := \max_{\mathbf{g} \in \mathbb{R}^n, \|\delta\| \leq 1} \mathbf{g}^\top \delta.$$

In this particular case, one can use the property that the dual of $\|D^{-1} \cdot\|_\infty$ is given by $\|D \cdot\|_1$. However, since the norms are not differentiable, this problem is not smooth. To obtain a differentiable problem, slack variables have to be introduced in order to obtain a smooth formulation [8]. By applying the linearization a derivative in the cost function has been introduced, causing the need of second order derivatives when using the SQP algorithm for optimization.

To define a robust optimization problem with stochastic quantities (UQ Rob Opt), the standard deviations have to be taken into account, see e.g. [9],

$$\min_{\mathbf{p} \in \mathbb{R}^3} J_3(\mathbf{p}(\omega)) := \mathbb{E}[J_1(\mathbf{p}(\omega))] + \lambda \text{std}[J_1(\mathbf{p}(\omega))], \quad (8a)$$

subject to

$$G_3(\mathbf{p}(\omega), \mathbf{a}(\mathbf{p}(\omega))) := \mathbb{E}\left[G_1^{(i)}(\mathbf{p}(\omega), \mathbf{a}(\mathbf{p}(\omega)))\right] \\ + \lambda \text{std}\left[G_1^{(i)}(\mathbf{p}(\omega), \mathbf{a}(\mathbf{p}(\omega)))\right] \leq 0. \quad (8b)$$

where λ is a weighting factor, similar to D in (2). To calculate the stochastic quantities the sampling is done by using generalized polynomial chaos (gPC) and Monte Carlo (MC). For gPC a tensor grid of $5 \times 5 \times 5$ is constructed and a Gauß-Legendre quadrature is applied [10]. For MC 5000 random samples are generated, which leads to an error of less than 0.1% on the expectation value of E_0 .

The idea of linearization used for D Rob 1 can also be applied to the stochastic quantities used in UQ Rob Opt. One retrieves (UQ Lin Opt)

$$J_4(\bar{\mathbf{p}}) = \mathbb{E}[J_1(\bar{\mathbf{p}} + \delta')] + \lambda \text{std}[J_1(\bar{\mathbf{p}} + \delta')] \\ \approx J_1(\bar{\mathbf{p}}) + \lambda \|\text{std}[\delta'] \circ \nabla_{\bar{\mathbf{p}}} J_1(\bar{\mathbf{p}})\|_2, \quad (9a)$$

where \circ depicts the elementwise product. If one chooses $\lambda = \frac{D}{\text{std}[\delta']}$, one obtains an expression equivalent to (7a), when considering the 2-norm (D Rob 2). The constraints are

$$G_4(\bar{\mathbf{p}}, \mathbf{a}(\bar{\mathbf{p}})) = \mathbb{E}\left[G_1^{(i)}(\bar{\mathbf{p}} + \delta', \mathbf{a}(\bar{\mathbf{p}} + \delta'))\right] \\ + \lambda \text{std}\left[G_1^{(i)}(\bar{\mathbf{p}} + \delta', \mathbf{a}(\bar{\mathbf{p}} + \delta'))\right] \\ \approx G_1^{(i)}(\bar{\mathbf{p}}, \mathbf{a}(\bar{\mathbf{p}})) \\ + \lambda \|\text{std}[\delta'] \circ \nabla_{\bar{\mathbf{p}}} G_1^{(i)}(\bar{\mathbf{p}})\|_2. \quad (9b)$$

IV. DISCUSSION

Starting from the initial geometry [2] ($p_1 p_2 = 133 \text{ mm}^2$ and $E_0 = 30.370 \text{ V}$), all approaches reduce the size of the PM roughly by a factor of two while maintaining E_d . The influence of Δ is visualized in Fig. 1. For Δ tending to zero all methods converge to the result of D Opt. The numerical results for D Rob 2 and UQ Lin Opt coincide since both approaches are mathematically equivalent as will be discussed in the full contribution. This indicates that using linearization for robust optimization in the deterministic and in the stochastic setting is equivalent. The results using robust optimization in the UQ and deterministic setting do differ. D Rob 1 is a more pessimistic scenario since it mitigates the worst case. Using the second moment, more information is incorporated during optimization. This leads to more optimistic results, because rare events are neglected.

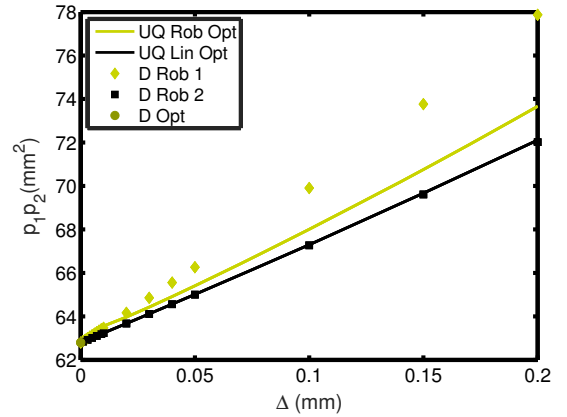


Fig. 1: Optimized size as a function of the deviation Δ .

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